

Theory of Pre-Asymptotic Effects in Weak Inclusive Decays

*Talk at the Workshop "Continuous Advances in QCD"**

February 18 – 20, 1994, TPI, Univ. of Minnesota, Minneapolis

M.A. Shifman

Theoretical Physics Institute, Univ. of Minnesota, Minneapolis, MN 55455
e-mail address: SHIFMAN@VX.CIS.UMN.EDU

Abstract

I give an introduction to the theory of preasymptotic effects based on the systematic OPE/HQET expansion in $1/m_Q$ where m_Q is the heavy quark mass. The general idea is explained in two most instructive examples, with an emphasis on pedagogical aspects. Some important results of the last year are reviewed. In discussing the issue of the quark-hadron duality, one of the basic ingredients of the theory, I prove that the operator product expansion *per se* is an asymptotic expansion. The behavior of the high order terms in this expansion determines the onset of duality and the accuracy of the duality relations. The factorial divergence of the high-order terms in OPE implies a sophisticated analytical structure in the α_s plane, with terms of the type $\exp[-\exp(1/\alpha_s)]$.

*Extended version

1 Introduction

Although the weak inclusive decays of heavy flavor hadrons are driven by weak interactions the theory of pre-asymptotic effects is an important applied branch of QCD. The theory has been gradually crystallizing from the eighties, with a decisive breakthrough achieved recently, in the last two or three years. I will try to summarize what we know and what has to be done in future.

My talk is preceded by that of Dr. Neubert covering, in part, the same topic. Therefore, I will avoid those aspects that have been already discussed. My task is two-fold. First, I will sketch a general picture of the phenomena lying behind the pre-asymptotic corrections. It is remarkable that the most essential aspects of the theory that I am going to describe have very close parallels with the well-known phenomena from general physics and/or quantum mechanics, for instance, the Doppler effect, the Mössbauer effect, etc. It is a pleasure to reveal these analogies by translating from the language of the QCD practitioners to that of general physics.

As it often happens, practical needs make us think of deep theoretical questions. One of such issues, closely related to the theory of preasymptotic effects, is the onset of duality in QCD. When and how it sets in? The answer to these questions is important for understanding the accuracy of numerous predictions of our theory. Some ideas relevant to this issue will be discussed in the end of my talk.

The second task is to present the theory, as it exists now, in a historical perspective. The majority of the theorists active in the field are so young that they perceive 10-year old results as an ancient history. Therefore it would be in order to remind that both, the ideas we rely upon now and the relevant mathematical apparatus, have emerged in the eighties being, in turn, a natural continuation of the previous work.

2 Formulation of the problem

Heavy flavor hadrons H_Q contain a heavy quark Q (practically one can speak of the b quark and, to a lesser extent, of the charmed quark) plus a light cloud built from light quarks (antiquarks) and gluons. The heavy quark Q experiences a weak transition. The nature of this transition is of no concern to us here. It can be a radiative transition, like $b \rightarrow s\gamma$, semileptonic decay like $b \rightarrow cl\nu_l$ or a non-leptonic decay to lighter quarks, e.g. $b \rightarrow c\bar{u}d$ (Fig. 1). It is assumed that at short distances the amplitude is known from the electroweak theory. The task is to calculate the decay rate of the hadron H_Q and other decay characteristics: the energy spectra, the average invariant mass of the hadronic state produced, etc.

Asymptotically, if the heavy quark mass m_Q is infinitely large, the presence of the light cloud around Q plays no role. One can treat Q as a free heavy quark decaying into free final quarks. This is the famous parton approximation of the pre-historic era, the seventies. Having accepted this prescription we, of course, immediately calculate whatever we are asked to. Moreover, the result is obviously universal, independent of the structure of the light cloud. In particular, all lifetimes are predicted to be the same,

$$\Gamma(H_Q) = \Gamma(Q). \quad (1)$$

While this simple approach is certainly valid in the asymptotic limit $m_Q = \infty$, and the universality is the feature of this limit, it is totally unsatisfactory for down-to-earth

physics of the actual heavy quarks. Indeed, a single glance at the lifetime hierarchy in the charmed family shows how far we are from the asymptotic regime. The D^+ meson, the most long-living member of the family, has the lifetime exceeding that of Ξ_c^0 by more than an order of magnitude,

$$\tau(D^+)/\tau(\Xi_c^0) \sim 12. \quad (2)$$

Thus, one can hardly speak of pre-asymptotic corrections in this case – the “corrections” actually totally determine the decay pattern. In the b quark family they are expected to be more modest, but still theoretical control over the inclusive decays is impossible without calculating pre-asymptotic effects in a systematic manner. Their dynamical origin is quite obvious: interaction of the heavy quark Q and its decay products with the light cloud produces a series of terms suppressed by inverse powers of m_Q . (Perturbative QCD corrections emerging from the hard gluon exchanges and suppressed logarithmically, by powers of $\alpha_s(m_Q)$, can be trivially incorporated in the parton model and will not be touched upon here.)

3 Quarks and gluons versus general physics

Different dynamical mechanisms generate the $1/m_Q$ corrections. The essence of the phenomenon can be best explained by a transparent example familiar to everybody from the standard courses on quantum mechanics and nuclear physics. Assume we are interested in the nucleon β decay; the nucleon at hand is not free but is rather bound inside a nucleus. The nucleus contains several protons and neutrons glued by the nuclear forces and is surrounded by an electron shell (Fig. 2). The energy release in the β decay, ΔE , is certainly much larger than a typical binding energy of the electrons in the shell. Moreover, we will further assume – counterfactually – that $\Delta E \gg \epsilon_{nucl}$ where ϵ_{nucl} is the nucleon binding energy in the nucleus. Such nuclei do not exist in nature, but for our pedagogical purposes it is convenient to pretend that they do.

We start from a free neutron β decay, $n \rightarrow p e \bar{\nu}_e$, and then ask what distinguishes it from that of a bound nucleon. The answer is well-known: (i) the Pauli interference; (ii) the K capture (by proton); and, finally, (iii) the Fermi motion, i.e. the momentum spread of the decaying neutron. Let us discuss these phenomena in turn.

The Pauli exclusion principle does not allow the electron produced to have the same momentum as that of the electrons in the shell. Typically the electron momentum is of order ΔE , much larger than that of the bound electrons, and interference does not occur. In a “corner” of the phase space, however, the electron produced is soft implying a strong interference. This happens rarely; the effect is clearly suppressed by inverse powers of ΔE .

The inverse β decay due to the K capture is simply impossible for a free nucleon. The power suppression here is due to the fact that the amplitude is proportional to $\psi(0)$, the bound electron wave function at the origin.

Finally, the momentum spread of the bound nucleon – a typical bound state effect – results in a smearing of the momentum distribution of the produced electron. If the decaying neutron has the momentum parallel to that of the electron the latter is accelerated while for the antiparallel configuration it is decelerated. As a result, the total decay rate is also distorted, although the distortion is parametrically small provided that $\epsilon_{nucl}/\Delta E \ll 1$.

The description above is qualitative. Quantitative analysis in the case at hand presents no difficulties. The standard quantum-mechanical methods and the knowledge of the wave

functions (of the electrons in the shell as well as the nucleons in the nucleus) solves the problem. For instance, to take into account the Pauli interference we just antisymmetrize the total wave function with respect to the coordinates of the electron produced and that in the shell. In this way we get the interference term automatically. To include the Fermi motion the neutron momentum distribution is convoluted with the amplitude of the free decay.

The pre-asymptotic effects in the inclusive decays of heavy flavors are perfectly similar in nature. The decay products of the heavy quark can interfere with the quarks from the light cloud. The heavy quark can capture a light one from the cloud. Finally, the “Fermi motion” of Q is also there: even if H_Q is at rest Q has a non-vanishing spatial momentum of order of Λ_{QCD} . What is drastically different, however, is the fact that the wave function approach is useless in QCD.

Indeed, the light cloud surrounding Q is a messy dynamical system composed of an infinite number of degrees of freedom, quark and gluon, and any attempt to reduce its dynamics to quantum mechanics of a finite number of particles is doomed to failure from the very beginning.

Does this mean that no adequate theory can be developed? The question needs no answer since the answer has been already spelled out. A systematic approach does exist, but one has to pay a price. If in the quantum-mechanical case the effects listed above are tractable for arbitrary values of ΔE – the requirement $\Delta E \gg \epsilon_{nucl}$ was actually unnecessary – in the heavy flavor decays the theory is based on the expansion in Λ_{QCD}/m_Q ; practically only the first few terms are calculable. Still, even this limited progress in the quantitative description of the inclusive decays is of great practical value and leads to a remarkably rich phenomenology.

4 Formalism. A little bit of history

As usual in practical problems from quantum chromodynamics the only tool existing today for dealing (analytically) with non-perturbative effects is the Operator Product Expansion (OPE) [1]. The general idea of OPE was formulated by Wilson even before QCD and is very simple. Whatever amplitude is considered all contributions to this amplitude must be systematically sorted out into two classes: hard contributions and soft ones. One introduces an auxiliary parameter, μ , (normalization point); if characteristic frequencies in the given fluctuation are higher than μ this fluctuation is to be ascribed to the hard part and determines the coefficient functions. In QCD the coefficient functions are explicitly calculable provided that μ is chosen appropriately,

$$\alpha_s(\mu)/\pi \ll 1.$$

(Warning: one should not think that the coefficient functions are determined exclusively by perturbation theory; in principle, non-perturbative contributions may also be present, for instance, those from small-size instantons. What is important is the fact that the weak coupling regime is applicable to the coefficient functions. Practically, in the vast majority of problems, the coefficient functions are given by perturbation theory to a good approximation. This approximate rule is sometimes called *the practical version of OPE* [2].)

Soft fluctuations are referred to the matrix elements of local operators appearing in OPE. Their contribution is not analytically calculable in the present-day QCD. The corresponding

matrix elements can be parametrized, however, or extracted from other sources by using symmetry arguments, QCD sum rules or lattice calculations, etc. All dependence on large parameters, such as m_Q , resides in the coefficient functions.

The general strategy outlined above has its peculiarities in application to the inclusive weak decays. The standard procedure is formulated in the euclidean domain where the distinction between the short and large distances (high and low frequencies) causes no doubts. Kinematics of the heavy flavor decays is essentially Minkowskian, however; the large expansion parameter is the energy release rather than the Euclidean off-shellness. The light quarks, once produced in the Q decay, stay forever – thus, formally, we have to deal with the infinite time intervals. Large distance dynamics manifests itself in full in the exclusive decays. At the same time, in the inclusive description, after summing over all possible channels and certain energy integrations, the results for the coefficient functions depend only on the short distance dynamics. This assertion, known as the quark-hadron duality, can be justified by means of an analytic continuation in the complex plane in auxiliary momenta. The accuracy of the quark-hadron duality is exponential in large momenta relevant to the given problem. I will return to this point, the exponential accuracy, later on.

Technically, in order to launch the OPE-based program one has to consider the so-called transition operator \hat{T} related to the modulus squared of the amplitude of interest [3], rather than the decay amplitude *per se*,

$$\hat{T}(Q \rightarrow f \rightarrow Q) = i \int d^4x \{ \mathcal{L}_W(x), \mathcal{L}_W^\dagger(0) \}_T, \quad (3)$$

where \mathcal{L}_W is the short-distance weak lagrangian governing the transition $Q \rightarrow f$ under consideration. The finite state f can be arbitrary; Fig. 3 displays, as a particular example, the graph pertinent to the semileptonic transition $Q \rightarrow ql\nu_l$ where q stands for the light quark. This graph is rather symbolic; one should keep in mind that only the skeleton lines are depicted, and all these lines are actually submerged in a soft gluonic medium coming from the light cloud in H_Q .

Next, we use the fact that the momentum operator of the heavy quark Q contains a large c -number part,

$$\mathcal{P}_\mu = m_Q v_\mu + \pi_\mu \quad (4)$$

where v_μ is the four-velocity of the heavy *hadron* H_Q , π_μ is the residual momentum operator,

$$[\pi_\mu, \pi_\nu] = iG_{\mu\nu},$$

and $G_{\mu\nu}$ is the gluon field strength tensor corresponding to the “background” gluon field in the light cloud. The presence of the large mechanical part in the Q quark momentum operator ensures that all lines encircled by a dashed line on Fig. 3 can be treated as hard. This block, thus, shrinks into a point; it determines the coefficient functions in front of local operators built from soft lines. The set of the soft lines includes the gluon field, light quark fields and those describing Q .

Once the general idea is understood it does not take much effort to calculate \hat{T} in the form of the Wilson expansion,

$$\hat{T} = \sum_n C_n \mathcal{O}_n, \quad (5)$$

where the operators \mathcal{O}_n are ordered according to their dimensions (or, under certain circumstances, according to their twists). The coefficients C_n are proportional to the corresponding powers of $1/m_Q$, modulo calculable logarithms.

At the next stage we average the transition operator over the hadronic state H_Q , i.e. find $\langle H_Q | \hat{T} | H_Q \rangle$. It is at this stage that the internal structure of H_Q enters in the analysis. The matrix elements $\langle H_Q | \mathcal{O}_n | H_Q \rangle$ parametrize the dynamical properties of the light cloud and its interaction with the heavy quark Q . They substitute the wave function of the quantum-mechanical treatment, the closest substitute one can possibly have in QCD.

Finally, at the very last stage, to make contact with measurable quantities we take the imaginary part of the matrix element $\langle H_Q | \hat{T} | H_Q \rangle$. In the example suggested above (Fig. 3) this imaginary part is proportional to the total semileptonic decay rate,

$$(M_{H_Q})^{-1} \text{Im} \langle H_Q | \hat{T} | H_Q \rangle = \Gamma_{SL}. \quad (6)$$

If the lepton energy is not integrated over but is held fixed the imaginary part of the transition operator is proportional to the energy spectrum, etc.

Analysis of the weak inclusive decays along these lines has been suggested in the mid-eighties when some operators in the expansion (5) relevant to the total lifetimes in the families of charm and beauty have been considered [3]. Many elements of what is now called the Heavy Quark Effective Theory (HQET) [4, 5] can be found in these works. Further progress has been deferred for a few years by an observation of certain difficulties in constructing OPE in the Minkowski domain [6]. The difficulties turned out to be illusory and are explained by the fact that contributions discussed in Ref. [6] were not fully inclusive. Once this has been realized [7] the last obstacle disappeared and a breakthrough in the theory of pre-asymptotic effects became inevitable. First systematic results – the leading $1/m_Q^2$ corrections to the parton-model predictions – have been obtained shortly after [8, 9]. While the works of the eighties passed essentially unnoticed fusion of similar ideas, which took place, independently, in the very beginning of the nineties, resulted in HQET in its present formulation [4, 5]. HQET provided a simple and convenient language for discussing the $1/m_Q$ expansions, in particular, in connection with the semileptonic decays. General aspects of the HQET/OPE approach to the semileptonic inclusive decays of heavy flavors have been discussed in Ref. [10]. The authors of this important work were not familiar with the previous development.

The current state of the theory of pre-asymptotic effects is characterized by a rapid expansion in the practical spheres. The $1/m_Q$, $1/m_Q^2$ and even $1/m_Q^3$ corrections are found for probably all conceivable quantities measurable in the inclusive decays of H_b and H_c . An incomplete list of works published in the last year and devoted to this subject includes a dozen of papers [11-22]. What remains to be done to complete the picture? The initial excitement, quite natural in time of the explosion of the theoretical predictions based on first principles, should give place to an every-day routine of extracting reliable and model-independent numbers from experimental data. This aspect – relation to experiment – will not be touched upon in my talk. Instead, I will demonstrate in more detail how the theory works in two problems which seem to be most instructive.

5 Total semileptonic widths. CGG/BUV theorem

The inclusive semileptonic decay rate is probably the simplest and the cleanest example. Although the parton-model analysis (including the first radiative QCD correction) is known

for more than a decade [23] the leading non-perturbative correction was calculated only in 1992, a pilot project [8] which opened the modern stage and was supplemented later by calculations of the energy spectrum and other distributions.

If one neglects the hard gluon exchanges, the transition operator is given by the diagram depicted on Fig. 4. The weak Lagrangian responsible for the semileptonic decay has the form

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} V_{Qq} (\bar{q} \Gamma_\mu Q) (\bar{l} \Gamma_\mu \nu), \quad (7)$$

where l is a charged lepton, electron for definiteness, and we assume for simplicity that the final quark q is massless ($\Gamma_\mu = \gamma_\mu(1 + \gamma_5)$). It is understood that the quark lines on Fig. 4 describe propagation in a soft gluon medium. The leptons are, of course, decoupled from gluons, and their lines are always treated as free.

The operator product expansion for the transition operator \hat{T} starts from the lowest-dimension operator, $\bar{Q}Q$ in the case at hand. In calculating the coefficient of this operator one disregards any couplings of the quark lines to the *soft* gluons from the cloud. The full perturbation theory resides in this coefficient, but – as I will explain shortly – it is wrong to say that the $\bar{Q}Q$ term in OPE just reproduces the parton-model prediction for Γ_{SL} ; actually it gives more.

There are no gauge and Lorentz invariant operators of dimension four [10] (and only Lorentz and gauge invariant operators can appear in the expansion for Γ_{SL}), and the first subleading term in OPE is due to the dimension 5 operator

$$\mathcal{O}_G = \bar{Q} \frac{i}{2} \sigma_{\mu\nu} G_{\mu\nu} Q. \quad (8)$$

This is the only Lorentz scalar dimension-5 operator bilinear in Q .

The corresponding coefficient C_G can be calculated in a variety of ways, of course; the easiest one is based on the background field technique and the Fock-Schwinger gauge [24]. In this gauge the line corresponding to the final quark q remains free (provided that we limit ourselves to the operator \mathcal{O}_G), and the only reason why the dimension-5 operator appears at all is the fact that we use the exact Dirac equations of motion with respect to the heavy quark lines Q and \bar{Q} .

For those who are not familiar with the Fock-Schwinger gauge technique let me add a remark elucidating the above statement. Explicit calculation of the diagram of Fig. 4, where all quark lines are considered in the background gluon field (with the Fock-Schwinger gauge condition), yields the operator $\bar{Q}(0) \not{p}(p^2)^2 Q(0)$ where $p = i\partial$. Now, we would like to rewrite p in terms of the covariant derivative $\mathcal{P} = p + A$ and the gluon field strength tensor – which is perfectly trivial in the Fock-Schwinger gauge – and then take advantage of the fact that $\not{\mathcal{P}}Q = m_Q Q$. It is not difficult to see that in the linear in $G_{\mu\nu}$ approximation one can substitute

$$\bar{Q}(0) \not{p}(p^2)^2 Q(0) \rightarrow \bar{Q}(0) \not{\mathcal{P}}(\mathcal{P}^2)^2 Q(0).$$

Supplementing this expression by the equalities

$$\mathcal{P}^2 = \not{\mathcal{P}}^2 - \frac{i}{2} \sigma G \text{ and } \not{\mathcal{P}}Q = m_Q Q$$

we conclude that in the Fock-Schwinger gauge the operator $\bar{Q} \not{p}(p^2)^2 Q$ reduces to $m_Q^5 \bar{Q}Q - 2m_Q^3 \mathcal{O}_G$. In other words, calculating the diagram of Fig. 4 with free fermions we get, for free, both coefficients – the coefficient in front of $\bar{Q}Q$ and that in front of \mathcal{O}_G !

I bring my apologies for this brief technical digression intended to demonstrate the beauty of the Fock-Schwinger technique which, unfortunately, is still unfamiliar to many heavy quark practitioners. Now it is time to present the final prediction [8] for the inclusive semileptonic decay rate,

$$\Gamma(H_Q \rightarrow X_q l \nu_l) = \frac{G_F^2 m_Q^5 |V_{Qq}|^2}{192\pi^3} \times \left\{ \frac{\langle H_Q | \bar{Q}Q | H_Q \rangle}{2M_{H_Q}} - \frac{2}{m_Q^2} \frac{\langle H_Q | \mathcal{O}_G | H_Q \rangle}{2M_{H_Q}} + \mathcal{O}(m_Q^{-3}) \right\}. \quad (9)$$

Perturbative hard gluon corrections are omitted here.

This is almost the end of the story. To complete the analysis one needs to know the matrix elements of the operators $\bar{Q}Q$ and \mathcal{O}_G over the hadronic state H_Q of interest. These matrix elements, clearly, depend on the structure of the light cloud which is beyond our computational abilities at present. Fortunately, they are known independently!

Indeed, \mathcal{O}_G is the lowest-dimension operator responsible for the spin splittings – the mass difference between the vector and pseudoscalar mesons (e.g. B^* and B). Thus,

$$\frac{\langle B | \mathcal{O}_G | B \rangle}{2M_B} \equiv \mu_G^2 = \frac{3}{4}(M_*^2 - M^2) \quad (10)$$

where M_* and M are the B^* and B masses, respectively. The right-hand side of eq. (10) is well measured experimentally. As for $\langle H_Q | \bar{Q}Q | H_Q \rangle$, let us compare the operator $\bar{Q}Q$ with the current $\bar{Q}\gamma_0 Q$ whose matrix element over any Q flavored hadron is unity, of course (in the H_Q rest frame). We are lucky again. The difference $\bar{Q}Q - \bar{Q}\gamma_0 Q$ is due to the lower components of the bispinors, and for heavy quarks these are reducible to the upper components by means of the equations of motion expanded in $1/m_Q$. In this way we arrive at [8]

$$\bar{Q}Q = \bar{Q}\gamma_0 Q + \frac{1}{2m_Q^2} \mathcal{O}_G - \frac{1}{2m_Q^2} \mathcal{O}_\pi + \mathcal{O}(m_Q^{-4}), \quad (11)$$

where

$$\mathcal{O}_\pi = \bar{Q}\vec{\pi}^2 Q$$

has the meaning of the average spatial momentum squared of the heavy quark Q inside H_Q .

As you probably remember, I told you that all perturbation theory is hidden in the coefficient of the operator $\bar{Q}Q$. The parton-model result is recovered if one approximates the matrix element $(2M_{H_Q})^{-1} \langle H_Q | \bar{Q}Q | H_Q \rangle$ by unity. Now we see that this is actually the zeroth order approximation; the very same operator $\bar{Q}Q$ gives rise to $1/m_Q^2$ and higher-order terms, for instance,

$$\frac{1}{2M_B} \langle B | \bar{b}b | B \rangle = 1 + \frac{\mu_G^2}{2m_b^2} - \frac{\mu_\pi^2}{2m_b^2} + \dots \quad (12)$$

where μ_π^2 parametrizes the matrix element of \mathcal{O}_π ,

$$\mu_\pi^2 = (2M_{H_Q})^{-1} \langle H_Q | \bar{Q}\vec{\pi}^2 Q | H_Q \rangle.$$

Assembling all these results together we finally find [8]

$$\Gamma(B \rightarrow X_u l \nu_l) = \frac{G_F^2 m_Q^5 |V_{bu}|^2}{192\pi^3} \times$$

$$\left\{ 1 + \text{zero} \times \frac{1}{m_b} + \left[\frac{1}{2m_b^2}(\mu_G^2 - \mu_\pi^2) \right] - \left[\frac{2}{m_b^2}\mu_G^2 \right] + \dots \right\}. \quad (13)$$

where the expression in the first square brackets is due to the $1/m_b^2$ correction in the matrix element of $\bar{b}b$, the term in the second square bracket comes from the operator \mathcal{O}_G in OPE for Γ_{SL} , and the dots denote higher-order terms in $1/m_b$.

The μ_π^2 part in the first square bracket has a very transparent physical interpretation. This is nothing else than the quadratic Doppler effect reflecting the time dilation for the heavy quark decaying in flight. Indeed, in the rest frame of the decaying hadron the quark b has a spatial momentum prolonging its lifetime (diminishing Γ). The effect linear in this momentum drops out because of averaging over angles, while the quadratic effect survives. It is quite obvious, from the first glance, that the coefficient of μ_π^2 in eq. (13) is exactly as it is expected to be in this picture.

A remarkable fact which I have especially emphasized in eq. (13) is the absence of the correction linear in $1/m_Q$ in the total semileptonic width. This practically important *theorem* (CGG/BUV theorem) is derived from two components – the absence of the dimension-4 operators in OPE and the absence of $1/m_Q$ corrections in the matrix elements of the operator $\bar{Q}Q$. The first circumstance has been noted in Ref. 10. The authors of this work believed, however, that linear in $1/m_Q$ terms may appear in $(2M)^{-1}\langle H_Q|\bar{Q}Q|H_Q\rangle$. The non-renormalization theorem for $(2M)^{-1}\langle H_Q|\bar{Q}Q|H_Q\rangle$ at the $1/m_Q$ level has been established in Ref. 8 completing the proof of the theorem.

6 The impact of the heavy quark motion in H_Q

Even if the Q flavored hadron is nailed at the origin so that its velocity vanishes the heavy quark Q moves inside the light cloud, its momentum being of order Λ_{QCD} . This is the QCD analog of the Fermi motion of the nucleons in the nuclei. It is quite clear that this motion affects the decay spectra. Say, if the “primordial” heavy quark momentum is parallel to that of the lepton pair, the pair gets boosted. A distortion of spectra may lead to corrections in the decay rate. It is equally clear that this effect is pre-asymptotic (suppressed by inverse powers of m_Q): while typical energies of the decay products are of order m_Q a shift due to the heavy quark motion is of order Λ_{QCD} .

A few models engineered to describe this effect at a purely phenomenological level exist in the literature for years (e.g. Ref. 25). Now one can finally approach the issue scientifically, backed up by a solid QCD-based analysis. The theory of this phenomenon is very elegant. I would like to demonstrate basic points in a simplest example, $Q \rightarrow q\gamma$ transition. Moreover, to avoid inessential technicalities (which can be readily worked out, though) I will neglect the quark and photon spins, assuming that both fields, Q and q are spinless. As for the mass of the final quark m_q it will be treated as a free parameter which can vary from zero almost up to m_Q . (For our approach to be valid we still need that $\Delta m \equiv m_Q - m_q \gg \Lambda_{QCD}$ although the mass difference may be small compared to the quark masses. As a matter of fact, this *small velocity* (or SV) limit [26] is very interesting and instructive, and I will return to its discussion later on.)

Thus, for pedagogical purposes we will consider, following Ref. 17, a toy model,

$$\mathcal{L}_\phi = h\bar{Q}\phi q + \text{h.c.} \quad (14)$$

where ϕ is a toy “photon” field and h is a coupling constant. To begin with, it is convenient to put the final quark mass to zero. At the level of the free quark decay the photon energy is fixed by the two-body kinematics, $E_\phi = m_Q/2$. In other words, the photon energy spectrum is a monochromatic line at $E_\phi = m_Q/2$ (Fig. 5). On the other hand, in the actual hadronic decays the kinematical boundary of the spectrum lies at $M_{H_Q}/2$. Moreover, due to multiparticle final states (which are, of course, present at the level of the hadronic decays) the line will be smeared. In particular, the *window* – a gap between $m_Q/2$ and $M_{H_Q}/2$ – will be closed (Fig. 6). There are two mechanisms smearing the monochromatic line of Fig. 5. The first is purely perturbative: the final quark q can shake off a hard gluon, thus leading to the three-body kinematics. The second mechanism is due to the “primordial” motion of the heavy quark Q inside H_Q and is non-perturbative. Only the second mechanism will be of interest for us here. The theory of the line shape in QCD resembles that of the Mössbauer effect.

Thus, the question to be answered is: “how can one translate an intuitive picture of the Q quark primordial motion inside H_Q in QCD-based theoretical predictions, not just plausible models?” The OPE/HQET approach is applicable here in full, with a few additional – rather transparent – technical elements.

The basic object of consideration is again the transition operator (3) (with \mathcal{L}_W substituted by \mathcal{L}_ϕ). Since we are interested in the energy spectrum the “photon” momentum q must be fixed.

Then in the approximation of Fig. 7 (no hard gluon exchanges) the transition operator takes the form

$$\hat{T} = - \int d^4x (x|\bar{Q} \frac{1}{(P_0 - q + \pi)^2} Q|0), \quad (15)$$

where the background field technique is implied and $(P_0)_\mu = m_Q v_\mu$. I remind the toy model with no spins for the quark and photon fields is taken for illustration; bellow a convenient shorthand notation

$$k = P_0 - q = m_Q v - q$$

will be used.

If \hat{T} is calculated in the operator product expansion the problem is solved since

$$\frac{d\Gamma}{dE} = \frac{h^2 E}{4\pi^2 M_{H_Q}} \text{Im} \langle H_Q | \hat{T} | H_Q \rangle. \quad (16)$$

To construct OPE we observe that, as in the previous problem of the total semileptonic widths, the momentum operator π corresponding to the residual motion of the heavy quark is $\sim \Lambda_{QCD}$ and the expansion in π/k is possible. Unlike the problem of the total widths, however, now in the interesting energy range k^2 is anomalously small, the expansion parameter is of order unity, and an infinite series of terms has to be summed up.

To elucidate this statement let us examine different terms in the denominator of the propagator,

$$k^2 + 2\pi k + \pi^2.$$

The effect of the primordial motion plays a key role in the window and in the adjacent region below it,

$$|E - (1/2)m_Q| \sim \overline{\Lambda} \quad (17)$$

where $\bar{\Lambda} = M_{H_Q} - m_Q$. It is quite trivial to find that in this domain

$$k_0 \sim |\vec{k}| \sim m_Q/2, \quad k^2 \sim m_Q \bar{\Lambda};$$

in particular, at the kinematical boundary (for the maximal value of the “photon” energy) $k_0 = M_Q/2$ and $k^2 = -m_Q \bar{\Lambda}$. Hence, inside the line

$$k^2 \sim k\pi \gg \pi^2.$$

In other words, when one expands the propagator of the final quark in eq. (15) in π , in the leading approximation all terms $(2k\pi/k^2)^n$ must be taken into account while terms containing π^2 can be omitted. The first subleading correction would contain one π^2 and arbitrary number of $2k\pi$ ’s, etc. For those who still remember the theory of deep inelastic scattering (DIS) of the late sixties and early seventies this may sound suspiciously familiar. Yes, you are right; the theory of the line shape in the radiative transition of the heavy quark into a massless one can be viewed as a version of the standard theory of DIS.

Thus, in this problem it is twist of the operators in OPE, not their dimension, that counts. Keeping only those terms in the expansion that do not vanish in the limit $m_Q \rightarrow \infty$ (analogs of the twist-2 operators in DIS) we get the following series for the transition operator [17]

$$\hat{T} = -\frac{1}{k^2} \sum_{n=0}^{\infty} \left(-\frac{2}{k^2}\right)^n k^{\mu_1} \dots k^{\mu_n} (\bar{Q} \pi_{\mu_1} \dots \pi_{\mu_n} Q - \text{traces}). \quad (18)$$

Traces are subtracted by hand since they are irrelevant anyway; their contribution is suppressed as $k^2 \pi^2 / (k\pi)^2 \sim \bar{\Lambda}/m_Q$ to a positive power. Another way to make the same statement is to say that in eq. (18) the four-vector k can be considered as *light-like*, $k^2 = 0$.

After the transition operator is built the next step is averaging of \hat{T} over the hadronic state H_Q . Using only the general arguments of the Lorentz covariance one can write

$$\langle H_Q | \bar{Q} \pi_{\mu_1} \dots \pi_{\mu_n} Q - \text{traces} | H_Q \rangle = a_n \bar{\Lambda}^n (v_{\mu_1} \dots v_{\mu_n} - \text{traces}) \quad (19)$$

where a_n are constants parametrizing the matrix elements. Their physical meaning will become clear momentarily. Right now it is worth noting that the term with $n = 1$ drops out ($a_1 = 0$). Indeed, $\langle H_Q | \bar{Q} \vec{\pi} Q | H_Q \rangle$ is obviously zero for spinless H_Q while π_0 through the equation of motion reduces to $\vec{\pi}^2 / (2m_Q)$ and is of the next order in $1/m_Q$. Disappearance of $\bar{Q} \pi_{\mu} Q$ means that there is a gap in dimensions of the relevant operators.

Let us write a_n ’s as moments of some function $F(x)$,

$$a_n = \int dx x^n F(x). \quad (20)$$

Then, $F(x)$ is nothing else than the primordial line-shape function! (That is to say, $F(x)$ determines the shape of the line before it is deformed by hard gluon radiation; this latter deformation is controllable in perturbative QCD). The variable x is related to the photon energy,

$$x = (2/\bar{\Lambda})[E - (1/2)m_Q].$$

If this interpretation is accepted – and I will prove that it is correct – it immediately implies that (i) $F(x) > 0$, (ii) the upper limit of integration in eq. (20) is 1, (iii) $F(x)$ exponentially falls off at negative values of x so that practically the integration domain in eq. (20) is limited from below at $-x_0$ where x_0 is a positive number of order unity.

To see that the above statement is indeed valid we substitute eqs. (19), (20) in \hat{T} ,

$$\langle H_Q | \hat{T} | H_Q \rangle = -\frac{1}{k^2} \sum_n \int dy F(y) y^n \left(-\frac{2\bar{\Lambda}kv}{k^2 + i0} \right)^n, \quad (21)$$

and sum up the series. The $i0$ regularization will prompt us how to take the imaginary part at the very end. In this way we arrive at [17]

$$\frac{d\Gamma}{dE} = -\frac{4}{\pi} \Gamma_0 \frac{m_Q E}{M_{H_Q}} \text{Im} \int dy F(y) \frac{1}{k^2 + 2y\bar{\Lambda}kv + i0} = (2/\bar{\Lambda}) \Gamma_0 F(x), \quad (22)$$

where the variable x is defined above and Γ_0 is the total decay width in the parton approximation. Corrections to eq. (22) are of order $\bar{\Lambda}/m_Q$.

Thus, we succeeded in getting the desired smearing: the monochromatic line of the parton approximation is replaced by a finite size line whose width is of order $\bar{\Lambda}$. The pre-asymptotic effect we deal with is linear in Λ_{QCD}/m_Q .

At this point you might ask me how this could possibly happen. I told you a few minutes ago that there is a gap in dimensions of operators in OPE – no operators of dimension 4 exist – and the correction to $\bar{Q}Q$ is also quadratic in $1/m_Q$ (the CGG/BUV theorem). No miracles – the occurrence of the effect linear in Λ_{QCD}/m_Q became possible due to the summation of the infinite series in eq. (18); no individual term in this series gives rise to Λ_{QCD}/m_Q .

To avoid misunderstanding it is worth explicitly stating that the primordial line function is *not* calculated; rather $F(x)$ is related to the light-cone distribution function of the heavy quark inside H_Q , *vis.* $\langle \bar{Q}(n\pi)^n Q \rangle$, $n^2 = 0$, or more explicitly

$$F(x) \propto \int dt e^{ixt\bar{\Lambda}} \langle H_Q | \bar{Q}(x=0) e^{-i \int_0^t n A(n\tau) d\tau} Q(x_\mu = n_\mu \tau) | H_Q \rangle, \quad (23)$$

where n is a light-like vector

$$n_\mu = (1, 0, 0, 1).$$

Moreover, this primordial function is not the one that will be eventually measured from $d\Gamma/dE$; the actual measured line shape will be essentially deformed by radiation of the hard gluons. I will say a few words about this effect later on.

Although we can not calculate $F(x)$ still our analysis is useful in many respects. First of all, a few first moments of $F(x)$ are known, as well is its qualitative behavior. Second, the QCD-based approach tells us that the “scientific” distribution function is one-dimensional, while in a naive model one would rather introduce a three-dimensional distribution in the Q spatial momentum. The reason is obvious: the momentum operator π_μ does not commute with itself, $[\pi_\mu, \pi_\nu] = iG_{\mu\nu}$, while $n\pi$ commutes. Finally, the logarithmic evolution of $F(x)$ is calculable in perturbative QCD.

For real quarks and photons, with spins switched on, calculation of the line shape is carried out in Refs. 17,18.

7 Varying the mass of the final quark

So far I was discussing the transition into a massless final quark. If we looked at the line shape presented on Fig. 5 more attentively, through a microscope, we would notice that a

smooth curve is obtained as a result of adding up many channels, specific decay modes. A typical interval in E that contains already enough channels to yield a smooth curve after summation is $\sim \bar{\Lambda}^2/m_Q$. In other words, in terms of the photon energy the duality interval is $\sim \bar{\Lambda}^2/m_Q$. Roughly one can say that the spectrum of Fig. 5 covers altogether $m_Q/\bar{\Lambda}$ resonance states produced in the H_Q decays and composed of q plus the spectator (I keep in mind here that the final hadronic state is produced through decays of highly excited resonances, as in the multicolor QCD). These states span the window between $m_Q/2$ and M_{H_Q} and the adjacent domain to the left of the maximum at $E = m_Q/2$.

It is very interesting to trace what happens with the line shape, the duality interval and the primordial distribution function as the final quark mass m_q increases [17].

The most obvious kinematical effect is the fact that the window shrinks. When we eventually come to the SV limit [26],

$$\Lambda_{QCD} \ll \Delta m \equiv m_Q - m_q \ll m_{Q,q}$$

it shrinks to zero. Indeed, in this limit the photon energy in the two-body quark decay, $\Delta m(1 + \Delta m(2m_Q)^{-1})$ differs from the maximal photon energy in the hadronic decay, $\Delta M(1 + \Delta M(2M_Q)^{-1})$, only by a tiny amount inversely proportional to m_Q (Δm and ΔM stand for the quark and meson mass differences, respectively). Simultaneously with the shrinkage of the window the peak becomes more asymmetric and develops a two-component structure (Fig. 8). The dominant component of the peak, on its right-hand side, becomes narrower and eventually collapses into a delta function in the SV limit. A shoulder develops on the left-hand side; the duality interval becomes larger and the number of the hadronic states populating the line becomes smaller. When we approach the SV limit the delta function component will consist of exactly one state – the lowest-lying q containing meson (the elastic component) while the shoulder (the inelastic component) will include several excitations and will stretch down to $E = E_{max} - \text{several units} \times \bar{\Lambda}$. The light-cone distribution function will evolve and will continuously pass into the *temporal* distribution function determining the form of the spectral line in the shoulder (Fig. 9).

This rather sophisticated picture, hardly reproducible in the naive models, stems from the same analysis [17]. The transition operator \hat{T} now has the form

$$\hat{T} = \frac{1}{m_q^2 - k^2} \sum_{n=0}^{\infty} \bar{Q} \left(\frac{2m_Q\pi_0 + \pi^2 - 2q\pi}{m_q^2 - k^2} \right)^n Q, \quad (24)$$

where q is the ϕ momentum. Notice that $2m_Q\pi_0 + \pi^2$ acting on Q yields zero (the equation of motion) and in the SV limit q must be treated as a small parameter,

$$q_0/m_Q \equiv E/m_Q = v \ll 1;$$

v is the spatial velocity of the heavy quark produced. Although v is small the inclusive description we develop is still valid provided that $\Delta m \gg \Lambda_{QCD}$.

In the zeroth order in q the only term surviving in the sum (24) is that with $n = 0$, and we are left with the single pole, the elastic contribution depicted on Fig. 9. This is the extreme realization of the quark-hadron duality. The inclusive width is fully saturated by a single elastic peak. What might seem to be a miracle at first sight has a symmetry explanation – the phenomenon is explained by the heavy quark symmetry which has been noted in this particular context in Refs. 26, 27. (The heavy quark symmetry is also called

the Isgur-Wise symmetry [28]. These authors generalized the observation of Refs. 26, 27 to arbitrary values of v .) On the other hand, the fact that the parton-model monochromatic line is a survivor of hadronization is akin to the Mössbauer effect.

If terms $\mathcal{O}(v^2)$ are switched on the transition operator acquires an additional part,

$$\hat{T}_{v^2} = \frac{4}{3} \bar{q}^2 \frac{1}{(m_q^2 - k^2)^3} \sum_{n=0}^{\infty} \left(\frac{2m_Q}{m_q^2 - k^2} \right)^n \bar{Q} \pi_i \pi_0^n \pi_i Q. \quad (25)$$

From this expression it is obvious that the shape of the v^2 shoulder is given [17] by the temporal distribution function $G(x)$ whose moments are introduced through the matrix elements

$$\langle H_Q | \bar{Q} \pi_i \pi_0^n \pi_i Q | H_Q \rangle = \bar{\Lambda}^{n+2} \int dx x^n G(x). \quad (26)$$

Alternatively, $G(x)$ can be written as a Wilson line along the time direction,

$$G(x) \propto \int dt e^{ixt\bar{\Lambda}} \langle H_Q | \bar{Q}(t=0, \vec{x}=0) \pi_i e^{-i \int_0^t A_0(\tau) d\tau} \pi_i Q(t, \vec{x}=0) | H_Q \rangle. \quad (27)$$

Intuitively it is quite clear why the light-cone distribution function gives place to the temporal one in the SV limit. Indeed, if the massless final quark propagates along the light-cone, for $\Delta m \ll m$ the quark q is at rest in the rest frame of Q , i.e. propagates only in time.

In terms of $G(x)$ our prediction for the line shape following from eq. (25) takes the form

$$\begin{aligned} \frac{d\Gamma}{dE} \propto & \left[1 - \frac{v^2}{3} \int \left(\frac{1}{y^2} + \frac{\bar{\Lambda}/E_{max}}{y} \right) G(y) dy \right] \delta(x) + \\ & \frac{v^2}{3} \left(\frac{1}{x^2} + \frac{\bar{\Lambda}/E_{max}}{x} \right) G(x), \end{aligned} \quad (28)$$

where $x = (E - E_{max})/\bar{\Lambda}$. The v^2 corrections affect both, the elastic peak (they reduce the height of the peak) and the shoulder (they create the shoulder). The total decay rate stays intact, however: the suppression of the elastic peak is compensated by the integral over the inelastic contributions in the shoulder. This is the Bjorken sum rule [29, 30]. It is important that we do not have to guess or make *ad hoc* assumptions – a situation typical for model-building – the theory itself tells us what distribution function enters in this or that case and in what particular way.

8 Inclusive semileptonic decays^[16,17,19,21]

The very same distribution functions that determine the line shape in the radiative transitions appear in the problem of the spectra in the semileptonic decays. In particular, in $b \rightarrow ul\nu$ we deal with $F(x)$. Their analysis is absolutely crucial if one addresses the behavior in the so-called end-point region (i.e. the region inaccessible in the parton model and close to it, an analog of the *window* in the radiative transitions).

Apart from trivial kinematical modifications – for instance, the occurrence of several structure functions – the only change is the expression for the variable x . More exactly, as

was shown above, for *massless* final quark in both cases, radiative transitions and semileptonic decays,

$$x = -k^2/(2\bar{\Lambda}kv),$$

but in the latter case $q^2 > 0$ while in the radiative transitions $q^2 = 0$, and this seemingly insignificant difference is practically important. Indeed, if $q^2 = 0$ the notion of scaling makes no sense – kinematically there is only one variable, the photon energy E , and $x \propto (E - E_{max})$. On the other hand, in the inclusive decays the structure functions depend, generally speaking, on two independent variables, k^2 and kv (or q^2 and q_0). The fact that the structure functions in $B \rightarrow X_u l \nu$ are actually functions of a single combination x is a very strong statement fully equivalent to the Bjorken scaling in deep inelastic scattering. Guesses about a scaling behavior in the inclusive semileptonic decays are known in the literature [31]. This is the first time ever we are able to say for sure what sort of scaling takes place, where it is expected to hold and where and how it will be violated.

In the most naive parton model, when gluons are not considered at all, $k^2 = 0$. The primordial heavy quark motion smears k^2 so that actually

$$k^2 \sim \bar{\Lambda}k_0. \quad (29)$$

I will sometimes refer to this domain as to “primordial”. In this domain the structure functions depend on

$$x = -\bar{\Lambda}^{-1}k^2/2k_0 = -\bar{\Lambda}^{-1}k^2/(k_0 + |\vec{k}|) = -\bar{\Lambda}^{-1}(k_0 - |\vec{k}|) \quad (30)$$

where in the denominator the difference between k_0 and $|\vec{k}|$ is neglected which is perfectly legitimate in the primordial domain (29). In the rest of the phase space this substitution is wrong, of course, but there the above scaling is not going to be valid anyway, so there is no need to bother. From eq. (30) we see that in the primordial domain the structure functions depend on a single light-cone combination

$$q_0 + |\vec{q}|,$$

rather than on q_0 and q^2 separately.

The primordial structure functions fall off – presumably exponentially – outside the primordial domain. The hard gluon emissions will populate the phase space outside this domain creating long logarithmic tails. The primordial part is buried under these tails. Therefore, outside the primordial domain one can not expect that the structure functions depend on the single combination $q_0 + |\vec{q}|$.

Perturbative corrections as well as those due to higher twist operators violate scaling in the primordial domain as well. If the latter are generically small the perturbative corrections in the end-point region, due to the Sudakov double log suppression, are expected to be large, so that practically in the measured structure functions the scaling feature will be implicit. Needless to say that perturbative violations of scaling are calculable.

I will not go into further details which are certainly important if one addresses the problem of extraction of V_{ub} from experimental data. Some of them are discussed in the current literature [17, 19], others still have to be worked out. Applications of the theory to data analysis is a separate topic going beyond the scope of the present talk. The only obvious remark on the visibility of the primordial motion in experimental data seems to be in order: the electron energy spectrum in $B \rightarrow X_u e \nu$ is probably the worst place for

studying $F(x)$. Indeed, here the primordial distribution function enters in an integrated form even in the absence of the hard gluon corrections. A smearing of $F(x)$ is unavoidable if we integrate over the neutrino variables, and then the effect of $F(x)$ is diluted, and is seen to a much lesser extent than in the radiative transitions. In the inclusive semileptonic decays a much better place to search is the double differential distribution in q_0 and q^2 .

9 $b \rightarrow cl\nu$

The formalism described above is fully applicable [17, 21], in principle, to the semileptonic inclusive transitions $b \rightarrow cl\nu$ where the final quark c can be also treated as heavy, although at the same time, $m_c^2 \ll m_b^2$. The ratio $m_c^2/m_b^2 \approx 0.07$ is a small parameter while $m_c^2/(\bar{\Lambda}m_b) \sim 1$. Technically, the situation becomes much more sophisticated, however, since the type of the distribution function reflecting the primordial motion of the b quark inside the B meson will depend now on the value of q^2 where q is the total momentum of the lepton pair.

If q^2 is small, $q^2 < m_c^2$, one recovers [21] the same light-cone function $F(x)$ as in the transition $b \rightarrow ul\nu$ or $b \rightarrow s\gamma$. Modifications are marginal. First, some extra terms explicitly proportional to m_c/m_b are generated in the structure functions due to the fact that $\mathcal{P} + m_c$ replaces \mathcal{P} in the numerator of the quark Green function. Moreover, if in the $b \rightarrow ul\nu$ transitions the scaling variable in the end-point domain is

$$x = \bar{\Lambda}^{-1}(q_0 + |\vec{q}| - m_b), \quad (31)$$

in the $b \rightarrow cl\nu$ transition it is shifted by a constant term of order 1,

$$x = \bar{\Lambda}^{-1}(q_0 + |\vec{q}| - m_b) + \frac{m_c^2}{\bar{\Lambda}m_b}. \quad (32)$$

Finally, subleading (higher-twist) terms in $b \rightarrow ul\nu$ generically give rise to corrections of order $k^2/k_0^2 \sim \bar{\Lambda}/m_b$. In the beauty-to-charm decays these corrections are $\mathcal{O}((\bar{\Lambda}m_b + m_c^2)/m_b^2)$.

As q^2 increases the corrections to the description [21] based on the light-cone function grow and eventually blow up when q^2 approaches its maximal value,

$$q_{max}^2 = (M_B - M_D)^2,$$

since here $k_0 \sim m_c$, not m_b . Practically this happens at

$$\sqrt{q^2} > M_B - 2M_D. \quad (33)$$

The domain (33) corresponds to the SV regime which I have already discussed, with fascination, in the toy example above. In the SV limit the velocity of H_c produced is small,

$$|\vec{v}|^2 \approx \frac{(M_B - M_D)^2 - q^2}{M_B M_D} \ll 1,$$

and the light-cone distribution function becomes irrelevant. The primordial motion of the b quark is described [17] in this regime by the temporal distribution function $G(x)$ where

$$x \approx \bar{\Lambda}^{-1}(q_0 - \Delta m),$$

Δm is the quark mass difference coinciding, to the leading order with $M_B - M_D$. The "corner" of the phase space where we find ourselves close to the SV limit is responsible for roughly 1/2 of all events in $B \rightarrow X_c l \nu$.

Thus, changing q^2 from zero to q_{max}^2 results in an evolution of the distribution function appearing in theoretical formulae for $d\Gamma(B \rightarrow X_c l \nu)$, from light-cone to temporal, through a series of intermediate distributions. The physical reason for this evolution is quite clear – what distribution function is actually measured depends on the parton-model velocity of the quark produced in the b decay.

In calculating the lepton energy spectrum, $d\Gamma/dE_l$, one needs to integrate over a range of q^2 . In doing so, we inevitably smear all these evolutionary stages. How exactly the effect of the primordial motion in $d\Gamma(B \rightarrow X_c l \nu)/dE_l$ looks like – nobody knows at present. An attempt to express this effect in terms of the light-cone function $F(x)$ has been recently reported [21]. One should expect that, given the actual values of the quark masses, the expression for $d\Gamma(B \rightarrow X_c l \nu)/dE_l$ obtained in this work will have corrections $\mathcal{O}(1)$.

10 The Roman distribution function

The moments of the light-cone distribution function $F(x)$ are related to the matrix elements of the operators (19). This infinite set of matrix elements codes information about the bound-state structure of H_Q . Needless to say that we are not able to calculate this infinite set in the present-day QCD; our possibilities are limited to a few first moments. Since the desire to get an idea of the primordial distribution function, to visualize $F(x)$, is naturally strong it seems reasonable to adopt a pragmatic attitude. Let us try to conjecture a qualitatively acceptable function, whose properties are compatible with the expected features of the primordial distribution function. This conjectured function will depend on several free parameters to be fixed from the moments we know.

No extensive guesswork is needed. Surprisingly, a phenomenological model existing for more than a decade will provide us with a quite adequate input for $F(x)$. I mean the so-called AC²M² model [25] for the "Fermi" motion of the heavy quark inside H_Q .

The physical essence of the model is very simple. It assumes that, say, the B meson is a compound system consisting of the heavy quark b and a spectator with a fixed mass m_{sp} (one of two free parameters of the model). The spectator component has a momentum distribution $\Phi(|\vec{p}|)$ where \vec{p} is a three-dimensional momentum of the spectator. The energy of the b quark is $E_b = M_B - (|\vec{p}|^2 + m_{sp}^2)^{1/2}$. The mass of the b quark then has to be equal to E_b through order p . Thus, we arrive at the notion of the floating b quark mass

$$m_b^f \approx M_B - (|\vec{p}|^2 + m_{sp}^2)^{1/2}, \quad (34)$$

technically a most important ingredient of the AC²M² model. The b quark with momentum $-\vec{p}$ decays into $ue\nu$ in flight; after averaging over

$$\Phi(|\vec{p}|) = \frac{4}{p_F^3 \sqrt{\pi}} e^{-\frac{p^2}{p_F^2}}$$

we obtain all relevant decay spectra (here p_F , a Fermi momentum, is a second free parameter of the model).

This model is routinely used for years in analyzing the experimental data on the inclusive semileptonic decays of B 's. Given all its naïveté it is quite remarkable that it still reproduces

essential features of the QCD-based theory of pre-asymptotic corrections. First of all, it has been demonstrated [32, 33] that the corrections of the first order in $1/m_b$ to the total decay rates are absent in this model, the CGG/BUV theorem. Second, in the primordial domain the correct scaling is recovered: the structure functions do indeed depend only on the combination (30) or (31).

It is not difficult to extract [33] the light-cone distribution function stemming from the AC²M² model,

$$F_{Rom}(x) = \frac{1}{\sqrt{\pi}} \frac{\bar{\Lambda}}{p_F} \exp \left\{ -\frac{1}{4} \left[\frac{p_F}{\bar{\Lambda}} \frac{\rho}{1-x} - \frac{\bar{\Lambda}}{p_F} (1-x) \right]^2 \right\}, \quad (35)$$

where *Rom* stands for Roman, and

$$\rho = (m_{sp}/p_F)^2.$$

The vanishing of the first moment implies

$$\frac{\bar{\Lambda}}{p_F} = \frac{\rho}{\sqrt{\pi}} e^{\rho/2} K_1\left(\frac{\rho}{2}\right), \quad (36)$$

where K_1 is the McDonald function. The Roman function satisfies the requirements one would expect from the primordial distribution function in the inclusive decays into massless quarks. First, it vanishes when one approaches $x = 1$ from below, so that no spectrum is generated beyond the physical kinematical boundary. Moreover, at the negative values of x it falls off exponentially once $|x| \gg 1$.

The Roman function depends on two parameters to be fitted, m_{sp} and p_F . Usually they are fitted from experimental data. Previously, when the theory of pre-asymptotic effects did not exist, there was no clear idea how to fit the data properly. New understanding calls for a new analysis, currently under way. Meanwhile it is instructive to see what can be said theoretically.

It is not difficult to show that the AC²M² model constrains the second moment from above, $\langle x^2 \rangle < (\pi - 2)/2$. The largest possible value, $\langle x^2 \rangle \sim 0.6$ is achieved if $\rho \rightarrow 0$. On the other hand, from the QCD sum rules one expects [34] $\langle x^2 \rangle \sim 0.5$ to 1. Therefore, if the AC²M² *ansatz* has chances to survive the value of ρ must be small. In other words, $m_{sp}^2 \ll p_F^2$, the spectator must be relativistic. In this case F_{Rom} is rather broad – its width is of order $\bar{\Lambda}$ – and very asymmetric, see Fig. 10. If ρ is indeed small, $p_F \approx (\sqrt{\pi}/2)\bar{\Lambda}$.

It may well happen that the Roman *ansatz*, being qualitatively reasonable in the $b \rightarrow u$ transition, will fail to quantitatively reproduce fine structure. With two free parameters it may turn out to be too restrictive. Then we will have to invent another clever *ansatz*, with three fit parameters.

I hasten to add that the AC²M² model does not reproduce correctly the $1/m_Q^2$ and higher effects, and it should not – it was not engineered for that purpose. What may be even more important, it gives no hints on the evolution of the light-cone distribution function towards the temporal one with increasing m_q . Therefore, the description based on this model is generally speaking inapplicable in the $b \rightarrow c$ transition; especially detrimental it becomes in the $B \rightarrow X_c e \nu$ decays at large q^2 , when we approach the SV limit. As I have mentioned a few minutes ago a large part of the phase space belongs to the SV regime. The strongest indication is the fact that two elastic modes, $B \rightarrow D e \nu$ and $B \rightarrow D^* e \nu$, yield more than 60 % of the total probability. In the SV regime the dominance of the elastic modes is absolute.

11 A few words about hard gluons

So far non-perturbative effects were my prime concern. This is the heart of the theory, its non-trivial part. To make contact with experiment, however, it is necessary to include a dynamically rather trivial effect due to the hard gluons, in the bremsstrahlung and in the loops. The bremsstrahlung corrections are most important in the case of the light final quarks and are moderate if the final quark is relatively heavy. Therefore, to make my point, I will consider, as a most instructive example [17], the radiative transition $b \rightarrow s\gamma$ ($B \rightarrow X_s\gamma$ at the hadron level). As you surely remember, in the parton approximation the photon spectrum is a monochromatic line at $E = m_b/2$. The primordial motion of the heavy quark, a non-perturbative effect, smears this line and generates a spectrum in the window and in the adjacent region below the window. The width of the line becomes of order $\bar{\Lambda}$. Outside the primordial domain whose size is $\sim \bar{\Lambda}$ the primordial motion has a negligible impact on the photon spectrum.

Even if the primordial motion is switched off, the line will be smeared anyway due to the hard gluon emission. A bremsstrahlung gluon can carry away a finite fraction of the accessible momentum, $\sim m_b/2$. This contribution produces events with the invariant mass of the hadronic state of order $m_b/2$; the corresponding photon energy spectrum is $\mathcal{O}(\alpha_s)$. The effect is calculable as a one-loop correction in the transition operator.

Apart from this trivial radiative tail the hard gluon emission modifies the peak itself, much in the same way as the electromagnetic radiative corrections modify the J/ψ peak in e^+e^- annihilation. There the natural width of the J/ψ meson is negligibly small, and the observed shape of the peak is determined by the radiative smearing due to the photon bremsstrahlung.

The gluon bremsstrahlung is characterized by large logarithms in the end-point domain; summation of these logarithms results in the Sudakov exponent. The Sudakov effect destroys the monochromatic line of the parton model. (It does not populate the window, though; it clearly remains empty in perturbation theory).

The peak at $E = m_b/2$ becomes less singular but still a singularity persists. The total probability remains the same as in the parton model but, if in the parton model it is all saturated in the peak, now a part of it is pumped into the adjacent tail. The area under the distorted peak turns out to be $\sim (\bar{\Lambda}/m_b)^{\epsilon_0}$ where ϵ_0 is a positive but relatively small number, $\epsilon_0 \sim 0.3$ [17].

For some reasons which I do not understand the problem of the Sudakov suppression in the end-point domain is perceived as something intractable: some authors say that the running coupling constant controlling these corrections blows up and no reliable calculations are possible. In principle, these questions have been studied in similar problems in the late seventies. To feel the running nature of α_s it is necessary to go beyond the double-log level. The answer is well-known at least at the first subleading level (with one logarithm lost): the coupling constant entering the Sudakov exponent is the running coupling normalized at k^2 where k^2 is the invariant mass squared of the hadronic state in the corresponding jet. In our case of the $b \rightarrow s\gamma$ transition $k^2 = (m_b v - q)^2$.

If so, the transformation of the parton-model delta function is easy to find. The Sudakov logarithms generate the following spectrum at the end-point:

$$d\Gamma/dE \propto \frac{1}{[1 - (2E/m_b)]^{1-\epsilon_0}} \theta\left(\frac{m_b}{2} - E\right) \quad (37)$$

where $\epsilon_0 = 8/(3b) \sim 0.3$ (b is the first coefficient in the Gell-Mann-Low function). This expression neglects subleading logarithms whose effect reduces to an overall factor $1 + \mathcal{O}(\epsilon_0)$. Thus, there is an uncertainty of order $\mathcal{O}(\epsilon_0)$ in the normalization of the end-point structure which must (and can) be eliminated by further calculations.

The final predictions for the spectral distributions, ready for comparison with experimental data, are obtained by superimposing the perturbative corrections with the non-perturbative effects I have discussed previously. The primordial shape function is smeared, of course, (in a well-defined way): its basic feature – a peak in the end-point domain – stays intact.

12 What is the heavy quark mass?

So far I have deliberately avoided any explicit definition of the heavy quark mass m_Q , as well as the related parameter $\overline{\Lambda}$, although both are the key parameters of the theory, appearing in virtually every expression. If you start thinking on this issue you will realize that the question is not that simple as it might seem at first sight and as it sometimes presented in the literature. The distinction between different definitions of mass is obviously a pre-asymptotic effect since the difference is of order Λ_{QCD} , a Λ_{QCD}/m_Q effect. Since we are aimed, however, at a systematic theory of pre-asymptotic effects ensuring accuracy $1/m_Q^2$ and higher we have to address the question of mass from scientific positions [35].

Isolated quarks do not exist in nature; therefore, the notion of an on-shell quark mass is meaningless. Heavy quarks are always surrounded by a light cloud. How much of it can one peel off?

The quantity most commonly used to parametrize m_Q is the so-called pole mass m_Q^{pole} . In perturbation theory the quark Green function is well defined in each (finite) order; the pole mass is, by definition, the position of the pole of the quark propagator to a given (finite) order in α_s . Thus, what people actually do is equivalent to separating out the non-perturbative component of the cloud from its perturbative (Coulomb) part. The perturbative component is supposed to be included in m_Q while the non-perturbative one to $\overline{\Lambda}$.

In purely perturbative calculations such a definition is quite acceptable, as well as many others. Moreover, it is even more convenient than others since the gauge invariance of m_Q^{pole} is explicit. This feature made it very popular since the mid-seventies (see e.g. Ref. 36). Important results of the perturbative calculations, such as the total semileptonic widths, are routinely expressed in terms of the pole mass.

What is acceptable in PQCD becomes absolutely unacceptable in the theory of non-perturbative effects. As a matter of fact, it is impossible to define m_Q^{pole} to the accuracy better than Λ_{QCD}/m_Q [35]. The discrimination “perturbative versus non-perturbative” contradicts the spirit and the letter of the Wilsonian operator product expansion and can not be carried out in a systematic way. The right thing to do is to separate out the low-frequency component of the cloud ($\omega < \mu$) from the high-frequency one ($\omega > \mu$), and include the high-frequency component in the definition of the quark mass. Then the low-frequency component will reside in $\overline{\Lambda}$. In other words, any scientific approach must deal with $m_Q(\mu)$, the running mass introduced at a distance μ below the would-be pole. As usual, the choice of the parameter μ is dictated by two opposite requirements: on one hand, to come as close to the soft non-perturbative domain as possible we want μ to be as small as possible; on

the other hand, to develop a consistent theory we must ensure that $\alpha_s(\mu)/\pi \ll 1$. The fact that both requirements can be met simultaneously is a highly non-trivial feature of QCD, established so far only at an empirical level. Practically $\mu = \text{several units} \times \Lambda_{QCD}$.

The fact that it is impossible to define m_Q^{pole} to the accuracy better than Λ_{QCD}/m_Q is seen in many different ways. One of the simplest arguments comes from the consideration of the divergence of perturbation theory in high orders due to the infrared renormalons [37-39]. Of course, nobody claims that the presence of the infrared renormalons is the only obstruction to introducing m_Q^{pole} , the *only uncontrollable* non-perturbative piece. But their occurrence is quite sufficient to kill any consistent calculation dealing with m_Q^{pole} and aimed at accuracy Λ_{QCD}/m_Q or better. The renormalon contribution gives an idea of the theoretical uncertainty which can not be eliminated, in principle, without introducing $m_Q(\mu)$.

Let us examine the perturbative series for the heavy quark mass. The renormalon divergence originates from the “bubble” insertions into, say, the one loop graph (Fig. 11). A standard expression for the one-loop renormalization can be written as

$$m_Q^{pole} - m_Q(\mu) = \frac{8\pi\alpha_s}{3} \int_{|\vec{k}| < \mu} \frac{d^3k}{(2\pi)^3} \frac{1}{\vec{k}^2}.$$

where μ is the normalization point of the running mass. Summing up all the bubbles we effectively substitute the bare coupling constant by the running one in the *integrand*,

$$m_Q^{pole} - m_Q(\mu) = \frac{8\pi}{3} \int_{|\vec{k}| < \mu} \frac{d^3k}{(2\pi)^3} \frac{\alpha_s(\vec{k}^2)}{\vec{k}^2}. \quad (38)$$

Now, the running gauge coupling is given by

$$\alpha_s(\vec{k}^2) = \frac{\alpha_s(\mu^2)}{1 - (b\alpha_s(\mu^2)/4\pi) \ln(\mu^2/\vec{k}^2)} = \alpha_s(\mu^2) \sum_{n=0}^{\infty} \left(\frac{b\alpha_s(\mu^2)}{4\pi} \ln \frac{\mu^2}{\vec{k}^2} \right)^n$$

where b is the first coefficient in the Gell-Mann-Low function. Substituting this expansion in eq. (38) and doing the k integration we arrive at a factorially divergent series

$$m_Q^{pole} - m_Q(\mu) = \frac{4\alpha_s(\mu)}{3\pi} \mu \sum_n n! \left(\frac{b\alpha_s(\mu)}{2\pi} \right)^n. \quad (39)$$

This series is not Borel-summable; truncating it at an optimal value of n , $n_0 \sim 2\pi/(b\alpha_s(\mu))$, we find that the intrinsic uncertainty in $m_Q^{pole} - m_Q(\mu)$ is

$$\Delta(m_Q^{pole} - m_Q(\mu)) \sim \frac{8}{3b} \mu \exp\left\{-\frac{2\pi}{\alpha_s(\mu)}\right\} \sim \frac{8}{3b} \Lambda_{QCD}. \quad (40)$$

(For a related discussion of the infrared renormalons in connection with HQET, with similar conclusions, see Ref. 40).

Eq. (38) counts the energy of the would-be Coulomb tail around a static color charge. The tail actually does not exist in the non-abelian theory; this is the reason explaining the intrinsic uncertainty (40).

The fact that there is no scientific way to define m_Q^{pole} to better accuracy does not mean, of course, that one can not construct a systematic expansion in $1/m_Q$. This expansion,

however, must be based on HQET/OPE and explicitly include the normalization point μ . Then the expansion parameter is actually $m_Q(\mu)$.

Although $m_Q(\mu)$ is not directly measurable it is related, nevertheless, to observable quantities. Let us elucidate the last point in more detail.

Assume that the structure functions in the semileptonic decay $B \rightarrow X_c l \nu$ are measured separately as functions of q_0 and $|\vec{q}|$; we are specifically interested in w_1 . (I will not bother you with precise definitions, w_1 is just one of five possible structure functions, see e.g. Ref. [13]). Assume furthermore that the measurement can be made at a fixed and small value of $|\vec{q}|$, i.e. $|\vec{q}| \ll M_D$. Then the plot of w_1 will schematically look as depicted on Fig. 12. Qualitatively it is similar to the plot of Fig. 8 where E is now replaced by q_0 ; a long radiative tail to the left of the shoulder was omitted on Fig. 8. This tail has nothing to do with the primordial motion, it is due to the hard gluon emission. The height of elastic delta function is 1, in certain units; a few conspicuous excitations in the shoulder have heights $\mathcal{O}(v^2)$ where $v = |\vec{q}|/M_D$ while the height of the radiative tail is further suppressed by $\alpha_s(q_{0max} - q_0)/\pi$.

Using our approach we can formally show [41] that the integral

$$\frac{1}{2\pi} \int (q_{0max} - q_0) dq_0 w_1 = \frac{\vec{q}^2}{2M_D^2} \bar{\Lambda} \quad (41)$$

modulo corrections of higher order in v and in Λ_{QCD} . The upper limit of integration is q_{0max} , the lower limit is formally zero.

In these terms the standard definition of a “universal constant” $\bar{\Lambda}$, and the corresponding pole mass, $m_b^{pole} = M_B - \bar{\Lambda}$, might be formulated as follows: (i) take the radiative perturbative tail to the left of the shoulder and extrapolate it all the way to the point $q_0 = q_{0max}$; (ii) subtract the result from the measured structure function w_1 ; (iii) integrate the difference over dq_0 with the weight function $(q_{0max} - q_0)/2\pi$. The elastic peak drops out and the remaining integral is equal to $(v^2/2)\bar{\Lambda}$.

When I rephrase the standard program that way its absurdity, from purely theoretical point of view, becomes evident. How can one define what the perturbative tail is at $(q_{0max} - q_0) \sim \Lambda_{QCD}$ even in the first order in α_s , to say nothing about high orders? Meanwhile, this is exactly what is implied in the pole mass: one includes in the pole mass, by definition, all perturbative corrections coming from all virtual momenta, up to the would-be mass shell (i.e. excludes them from $\bar{\Lambda}$).

What is the correct procedure? We *must* introduce a normalization point μ in such a way that all frequencies smaller than μ can be considered as “soft” or inherent to the bound state wave function; at the same time $\alpha_s(\mu)$ is still a well-defined notion and the perturbative expansion in $\alpha_s(\mu)/\pi$ makes sense. We then draw a line at $q_0 = q_{0max} - \mu$ (Fig. 12). The integral (41) taken in the limits from $q_{0max} - \mu$ to q_{0max} represents $(v^2/2)\bar{\Lambda}(\mu)$ modulo corrections of higher order in v and in Λ_{QCD} . The running mass is then defined as $m_b(\mu) = M_B - \bar{\Lambda}(\mu)$. Practically, if the radiative tail in w_1 is numerically suppressed compared to the height of the shoulder, the μ dependence of $\bar{\Lambda}$ may be rather weak.

13 The onset of duality. Divergence of OPE

In this part of my talk I will leave particular applications and address a general issue, the foundation of the theoretical approach developed. The theory of preasymptotic effects in

weak decays of heavy flavors is based on the Wilsonian operator product expansion. More exactly, we exploit a generalization of OPE relevant to the Minkowski kinematics. To justify the procedure we keep in mind a possibility of analytic continuation to the euclidean domain in some momenta. In the semileptonic decays we can analytically continue in the lepton pair momentum and consider the amplitudes of interest off all cuts, as it is done, for instance, in Ref. 10. In the general case it is always possible to introduce an auxiliary (complex) momentum flowing in (or out) the vertex generating the weak decay. The operator product expansion is unambiguously defined in the euclidean domain since here the separation of the large and short distance contributions is well-defined. Predictions in the Minkowski domain are then obtained through dispersion relations. The simplest prediction of this type is the calculation of the total e^+e^- annihilation cross section at high energies.

The coefficient functions in the original (euclidean) version of OPE are determined by the short distance dynamics. From the remark above it must be clear that the very same statement is valid for the physical observables in the Minkowski region provided they are fully inclusive and the predictions are understood in an integral sense, smeared over some energy range. A point-by-point prediction becomes possible only at sufficiently high energies, when the observables under consideration are already smooth and additional “by hand” smearing is not needed. The theory of the preasymptotic effects I have discussed above assumes that we are already inside this *duality domain*. The question of where the boundary of the duality domain lies in each particular process is one of the major practical questions in the non-perturbative QCD.

To make the point more transparent let us turn to the e^+e^- annihilation cross section where the kinematical conditions are much simpler than in the heavy flavor decays. If q is the total momentum of the pair, $q^2 > 0$ corresponds to the physical (Minkowski) region while negative values of q^2 (i.e. positive $Q^2 \equiv -q^2$) is the euclidean domain. The operator product expansion in the euclidean domain is the expansion of the current correlation function in the powers of $\alpha_s(Q)$ and Λ_{QCD}^2/Q^2 modulo logarithms of the same parameter. As well-known [37, 39] the perturbative series is factorially divergent and non-Borel-summable. The standard perturbation theory has to be amended.

It was amended [42] in the spirit of OPE – by means of introduction of the vacuum condensates,

$$\langle vac | \mathcal{O}_n | vac \rangle \sim \mu^{d_n} \exp \left[-\frac{2\pi d_n}{b\alpha_s(\mu)} \right] \times \log s, \quad (42)$$

where d_n stands for the dimension of the operator \mathcal{O}_n . This rather straightforward extension cures the divergence of the perturbative series; the exponential (in $1/\alpha_s$) terms appear explicitly. They are obviously non-analytic in α_s . These terms result in a complicated analytic structure in the complex α_s plane, with an infinite set of cuts [37].

The expansion for the current correlation function at $Q^2 > 0$ is translated in the prediction for the total cross section, $\sigma(s)$, where $s = q^2$. In the limit $s \rightarrow \infty$ this prediction is simple: an appropriately normalized ratio $R = \text{Const}(\sigma)$ is equal to unity (I assume for simplicity that the only quarks are the massless u and d). Of course, we are interested in deviations from this unity.

Every given term in the euclidean expansion results in a correction in $R(s)$ of a generic form

$$\sum a_n (\Lambda_{QCD}^2/s)^{d_n/2} \times \log s. \quad (43)$$

(Pure powers of $1/Q^2$ yield delta function of s and its derivatives invisible at $s \gg \Lambda_{QCD}^2$.)

We have calculated a variety of such terms in the case of the heavy flavor decays where the role of s is played by m_Q^2 . It is obvious that at $s \gg \Lambda_{QCD}^2$ to any finite order in eq. (43) the prediction for $R(s)$ is a smooth function of s .

The question is where the structures in $R(s)$ come from. The fact that the structures are there is clearly seen, say, in the multicolor QCD [43], with $N_c \rightarrow \infty$. In this limit $R(s)$ actually represents a comb of infinitely narrow peaks: the lowest one is due to the ρ meson and the rest are due to its radial excitations. Even if the number of colors is three the first excitation shows up in $R(s)$ as a noticeable structure although numerically $m_{\rho'}^2/\Lambda_{QCD}^2 \gg 1$.

The answer to the question above is in the behavior of the power expansion (43) in high orders [44] (or, better to say, in the behavior of the analogous euclidean expansion). The power series *per se* turns out to be asymptotic, it has factorially divergent coefficients. The reflection of this asymptotic nature of the expansion is the occurrence of the exponential terms of the type

$$\exp(-CQ^2/\Lambda_{QCD}^2)$$

in the euclidean domain. Their analytic continuation to the Minkowski region provides a structure in $R(s)$. They also set the rate of approach to the duality domain and determine the accuracy of the duality relations.

The factorial divergence of the power series may be called the divergence of the second generation. Accumulation of the 't Hooft singularities (43) produces a more sophisticated singularity [44]

$$\exp \left[-C e^{(4\pi b^{-1}/\alpha_s(Q))} \right]. \quad (44)$$

Unlike the 't Hooft singularities which, in principle, can be revealed by analyzing multi-loop Feynman graphs, the second-generation singularity (44) obviously remains undetectable diagrammatically (in the classical understanding of diagrammar).

The assertions above can be demonstrated in a few different ways. The simplest proof I know is as follows. Let us consider heavy-to-light quark currents

$$J_S = \bar{Q}q, \quad J_P = \bar{Q}\gamma_5 q; \\ J_1 = \frac{1}{2}(J_S + J_P) = \bar{Q}\frac{1}{2}(1 + \gamma_5)q, \quad J_2 = \frac{1}{2}(J_S^\dagger - J_P^\dagger) = \bar{q}\frac{1}{2}(1 + \gamma_5)Q. \quad (45)$$

Later on we will assume that $m_Q \rightarrow \infty$ and $m_q \rightarrow 0$. Note that J_2 is *not* hermitean conjugate to J_1 ; rather J_2 is a chiral partner to J_1^\dagger . Therefore, the correlation function

$$\Pi = i \int e^{ikx} dx \langle vac | T \{ J_1(x), J_2(0) \} | vac \rangle \quad (46)$$

vanishes in perturbation theory if m_q is put to zero. Thus, there is no uninteresting background, a feature which should be welcome, of course.

Formally we can write the correlation function (46) as a trace,

$$\Pi(k^2) = i \text{Tr} \left\{ \frac{1}{2}(1 + \gamma_5) \frac{m_q}{\mathcal{P}^2 - m_q^2 + (i/2)\sigma G} \frac{m_Q}{(\mathcal{P} + k)^2 - m_Q^2 + (i/2)\sigma G} \right\} \quad (47)$$

where \mathcal{P} is the momentum operator and the trace operation is defined with respect to this operator. Simplifying a little bit I will say that Π is the product of two quark Green's functions in the background gluon field; the averaging over this field is implied (Fig. 13a).

The $(1 + \gamma_5)$ projector in the currents $J_{1,2}$ ensures the “softness” of \mathcal{P} : only very small eigenvalues of $\mathcal{P}^2 + (i/2)\sigma G$ contribute since otherwise the explicit factor m_q in eq. (47) annihilates the result in the limit $m_q \rightarrow 0$ (compare with Ref. [45]).

Let us focus on the soft gluon fields discarding the effect of the hard gluon exchanges which is well-understood. First of all, we should choose our reference point, the external momentum k , in a most advantageous way. If $m_Q \rightarrow \infty$ it is clear that the optimal reference point lies “slightly” below threshold,

$$k_0 = m_Q - \epsilon, \quad \vec{k} = 0,$$

where ϵ scales with Λ_{QCD} , not m_Q .

Taking now the limit $m_Q \rightarrow \infty$, assuming the “softness” of \mathcal{P} (i.e. $\mathcal{P} \sim \Lambda_{QCD}$; this assumption is clearly not valid for graphs with the hard gluons which are disregarded anyway) and neglecting all terms suppressed by powers of $1/m_Q$ we find that

$$\Pi(\epsilon) = -i\text{Tr} \left\{ \frac{1}{4}(1 + \gamma_5) \frac{m_q}{\mathcal{P}^2 - m_q^2 + (i/2)\sigma G} \frac{1}{\epsilon - \mathcal{P}_0} \right\}. \quad (48)$$

The momentum operator \mathcal{P} is a perfect analog of the operator π from the previous sections. As a matter of fact, to be fully consistent, I should have replaced it by π ; and I will from now on. If $\epsilon \gg \Lambda_{QCD}$ one can expand eq. (48) in $1/\epsilon$. To write the expansion in a compact form it is convenient to introduce the vacuum expectation value of $\bar{q}q$ where q is the massless quark field. Then, according to [45],

$$\langle vac | \bar{q}q | vac \rangle = \lim_{m_q \rightarrow 0} (-i)\text{Tr} \left\{ \frac{m_q}{\pi^2 - m_q^2 + (i/2)\sigma G} \right\}. \quad (49)$$

Let us agree that in this section the angle brackets will denote the vacuum averaging. Then

$$\Pi(\epsilon) = \frac{1}{4\epsilon} \left[\langle \bar{q}q \rangle + \frac{1}{\epsilon^2} \langle \bar{q}\pi_0^2 q \rangle + \frac{1}{\epsilon^4} \langle \bar{q}\pi_0^4 q \rangle + \dots \right]. \quad (50)$$

The term with γ_5 in eq. (48) drops out since the vacuum state in QCD is P -even. What is more important – a key point in our analysis – is the disappearance of the odd powers of π_0 . The vacuum expectation values with the odd powers of π_0 all vanish due to the Lorentz invariance. Another way to see the absence of the odd powers of π_0 is to return to eq. (48) and to observe that it must stay intact under the change of the sign of the momentum operator, an obvious property of the trace operation. Certainly, one could have written the operator product expansion (50) directly by cutting the light-quark line on Fig. 13a (see Fig. 13b).

A question that immediately comes to one’s mind is how eq. (50) can possibly be correct. Indeed, let us assume that the expansion (50) is convergent. Then it defines a function which is odd under the reflection $\epsilon \rightarrow -\epsilon$. By no means one can allow the correlation function $\Pi(\epsilon)$ be odd!

At positive ϵ we are below the cut, and $\Pi(\epsilon)$ is analytic in ϵ . At negative ϵ , however, we sit right on the cut generated by the intermediate physical states produced by the currents $J_{1,2}$, and the correlation function $\Pi(\epsilon)$ develops an imaginary part, a discontinuity across the cut. Qualitatively we have a pretty good idea of how $\text{Im}\Pi$ looks like. I will dwell on this point later and now just state that on physical grounds the behavior of $\Pi(\epsilon)$ at positive and negative values of ϵ is absolutely different.

What is the solution of this apparent paradox?

Looking at the definition of the function Π and the currents (45) we observe that Π is actually a *difference* between the scalar and pseudoscalar channels,

$$\Pi \propto \langle J_S, J_S^\dagger \rangle - \langle J_P, J_P^\dagger \rangle.$$

Therefore, at first sight one might try to say that the spectral densities in these channels are identical and cancel each other in the difference so that $\Pi(\epsilon)$ has no imaginary part whatsoever. The expansion (50) itself tells us, however, that this conjecture is wrong. A brief reflection leads one to an oscillating imaginary part schematically depicted on Fig. 14. The oscillations are needed in order to kill all even terms in $1/\epsilon$ in the expansion of the dispersion representation,

$$\Pi(\epsilon) = \frac{1}{\pi} \int_{E_0}^{\infty} dE \frac{\text{Im } \Pi(E)}{\epsilon + E}. \quad (51)$$

The spectral density on Fig. 14 refers to the real QCD, with three colors (I remind that it is the difference between the spectral densities in the scalar and pseudoscalar channels and, therefore, it need not be positive). In the multicolor QCD, with $N_c \rightarrow \infty$, we would have, instead, two combs of infinitely narrow peaks sitting, back-to back, on top of each other. Let us defer for a while the discussion of how the spectral density evolves from $N_c = 3$ to $N_c = \infty$.

Returning to the expansion (50), the only logical possibility is to say that the coefficients of the $1/\epsilon$ expansion are actually factorially divergent in high orders, so that the expansion is asymptotic. Then a function defined by this expansion (plus the correct analytical properties, see eq. (51)) can well be essentially different at positive and negative values of ϵ , although superficially it produces an impression of an odd function of ϵ . (Examples of such functions are very well known in mathematics, and I will present one shortly.) In other words, the matrix elements $\langle \bar{q}(\pi_0)^{2n} q \rangle$ at large n behave as

$$\langle \bar{q}(\pi_0)^{2n} q \rangle \sim (-1)^n \langle \bar{q} q \rangle (\Lambda_{QCD}^2)^n C^{2n} (2n)!, \quad (52)$$

where C is a numerical constant. The value of this constant is crucial since it determines the onset of the duality regime.

The appearance of the factor $(-1)^n$ can be explained as follows. The vacuum expectation value (52) written as an euclidean integral has the form

$$\langle \bar{q}(\pi_0)^{2n} q \rangle = - \lim_{m_q \rightarrow 0} (-1)^n \text{Tr} \left[(\pi_0)^{2n} \frac{m_q}{\pi^2 + m_q^2 + (i/2)\sigma G} \right]_E, \quad (53)$$

where the conventions concerning the euclidean γ and σ matrices are borrowed from [46]. The subscript E marks the euclidean quantities. The expression in the square brackets is well-defined and is positive-definite.

Upon reflection it is not difficult to understand that the factorial growth of the condensates (52) with n is quite natural. Combining eqs. (50) and (52) on one hand with the dispersion relation (51), on the other, we conclude that the absolute value of $\text{Im}\Pi$ at large E is exponential in E ,

$$|\text{Im } \Pi| \sim \exp(-E/(C\Lambda_{QCD})) = \exp \left[-\frac{1}{C} e^{(2\pi b^{-1}/\alpha_s(E))} \right]. \quad (54)$$

The truncated operator product expansion (50) would predict $\text{Im}\Pi = 0$ at large E . Thus, the exponential in eq. (54) gives an estimate of the accuracy of the duality relations. The number C does indeed play a key role since the duality sets in at $E \gg C\Lambda_{QCD}$.

Speaking in terms of the analytical structure in the α_s plane we deal here with a sophisticated singularity of the type exponential of the exponential of $1/\alpha_s$!

I have no time to go into further details. Frankly speaking, quantitatively, not so many details are worked out. Let me make only two remarks.

If the number of colors increases from 3 to infinity the spectral density $\text{Im}\Pi$ at large E experiences a dramatic evolution. A smooth function depicted on Fig. 14 is converted into two combs of delta functions – scalar resonances have positive residues while pseudoscalars appear in $\text{Im}\Pi$ with negative residues; all residues are $\mathcal{O}(1)$. (Certainly, averaging over many resonances produces $\text{Im}\Pi \approx 0$.) This means that the terms in the operator product expansion (50) which are subleading in $1/N_c$ must be more singular in n , i.e. their divergence in high orders of OPE must be stronger. This aspect is not new, though; the very same situation takes place in the ordinary perturbation theory [47].

To wind up the topic let me give an example of a function whose properties are very similar to those we found for the correlation function (46). If $\psi(z)$ is the logarithmic derivative of the Γ function a schematic model for Π is as follows:

$$\Pi_{model} = \psi(\epsilon + \frac{1}{4}) - \psi(\epsilon - \frac{1}{4}) - \frac{1}{\epsilon - (1/4)}. \quad (55)$$

From the well-known properties of the ψ functions [48] we infer that Π_{model} is a sum of simple poles. All poles are situated at real negative values of ϵ , viz.

$$\epsilon = -\frac{1}{4}, -\frac{3}{4}, -\frac{5}{4}, -\frac{7}{4}, \dots \quad (56)$$

and all residues are ± 1 . The signs alternate: the first pole has negative residue, the second positive and so on. In other words, the analytic properties of Π_{model} are perfect – analytic everywhere except the points (56) corresponding to infinitely narrow resonances.

On the other hand, at *large positive* ϵ our model correlation function is expandable¹ in $1/\epsilon$ and, moreover, the expansion looks like an odd function,

$$\Pi_{model} = -\frac{1}{2\epsilon} \times \sum_{n=0}^{\infty} \frac{a_n}{(\epsilon^2)^n}, \quad (57)$$

where a_n are calculable coefficients. At large n they are related to the Bernoulli numbers B_n . As well-known, the latter grow as $B_n \sim (2n)!$ (see [48], page 23), and the coefficients a_n grow in the same manner. Thus we see that Π_{model} is a perfect example, it is exactly what we want to get in QCD dynamically.

14 Conclusions

Kolya Uraltsev recently reminded me about a remark with which Bjorken concluded his talk at *Les Rencontre de la Valle d'Aoste* in 1990 [29]. Bj said then "... within a year or two it is quite possible that the language used in describing heavy quark decay phenomenology

¹As a matter of fact, the asymptotic expansion (57) is valid everywhere in the complex plane except a small angle near the negative real semiaxis, see sects. 3.12 and 3.13 in [48].

will shift away from comparison of data with Model A or Model T, and instead be phrased in a language which deals with the importance of a correction of Type X or Type Y...". He was slightly inaccurate in two respects: elements of the QCD-based theory of preasymptotic effects had existed before 1990, and it took about four years – not one or two – to complete the first stage of the theory. In all the rest his anticipation came true – today any serious work related to the heavy flavor decays has to start from the $1/m_Q$ expansion and analysis of different corrections.

There is still a lot of work to be done. On the practical side, the $1/m_Q$ expansion has to be built at least up to the level of $1/m_Q^3$ in all problems of interest. We must create a catalog of the H_Q expectation values of relevant operators, much in the same way as the vacuum condensates were classified and estimated within the QCD sum rule method. The issue of the gluon radiative corrections has to be exhaustively worked out.

On the theoretical side, the most important question, the heart of the OPE/HQET theory, is understanding the quark-hadron duality: when it sets in, what deviations one may expect in any given process, etc. These questions are intimately related to the general structure of QCD at large distances and the color confinement mechanism. I do not exclude that attempts to answer these questions emerged from the needs of the applied QCD will help to QCD fundamentalists – those who still hope that an analytic solution of the (continuous) four-dimensional quantum chromodynamics is possible.

Acknowledgments I would like to thank my co-authors, I. Bigi, B. Blok, N. Uraltsev, A. Vainshtein and M. Voloshin, with whom I shared the pleasure of working on the theory of preasymptotic effects, for endless enthusiastic discussions of all topics relevant to the subject. Discussions with B. Grinstein, D. Gross, I. Kogan, A. Migdal, N. Nekrasov, M. Neubert, L. Randall and M. Wise are gratefully acknowledged.

This work was supported in part by DOE under the grant number DE-FG02-94ER40823.

References

- [1] K. Wilson, *Phys. Rev.* **179** (1969) 1499; *Phys. Rev.* **D3** (1971) 1818; Proc. 1971 Int. Symp. on Electron and Photon Interactions at High Energies, N. Mistry, Ed. (New York, 1972), p. 115;
K. Wilson and J. Kogut, *Phys. Reports* **12** (1974) 75;
A. M. Polyakov, 1970, unpublished.
- [2] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B249** (1985) 445 [reprinted in M. Shifman, Ed., *Vacuum Structure and QCD Sum Rules*, North-Holland, 1992, p. 161].
- [3] M. Shifman and M. Voloshin, 1982, see in V. Khoze and M. Shifman, *Uspekhi Fiz. Nauk* **140** (1983) 3 [*Sov. Phys. Uspekhi* **26** (1983) 387];
N. Bilić, B. Guberina and J. Trampetić, *Nucl. Phys.* **B248** (1984) 261;
M. Voloshin and M. Shifman, *Yad. Fiz.* **41** (1985) 187 [*Sov. Journ. Nucl. Phys.* **41** (1985) 120]; *ZhETF* **91** (1986) 1180 [*Sov. Phys. – JETP* **64** (1986) 698].
- [4] E. Eichten and B. Hill, *Phys. Lett.* **B234** (1990) 511;
H. Georgi, *Phys. Lett.* **B240** (1990) 447.
- [5] For recent reviews see e.g.
N. Isgur and M.B. Wise, in: *B Decays*, S. Stone, Ed. (World Scientific), Singapore, 1992;
H. Georgi, in Proceedings of the Theoretical Advanced Study Institute 1991, R.K. Ellis, C.T. Hill and J.D. Lykken, Eds. (World Scientific), Singapore, 1992;
M. Neubert, Preprint SLAC-PUB-6263, 1993 [to be published in *Phys. Reports*].
- [6] M. Voloshin, N. Uraltsev, V. Khoze and M. Shifman, *Yad. Fiz.* **46** (1987) 181 [*Sov. Journ. Nucl. Phys.* **46** (1987) 112].
- [7] I.I. Bigi and N.G. Uraltsev, *Phys. Lett.* **280B** (1992) 120.
- [8] I. Bigi, N. Uraltsev and A. Vainshtein, *Phys. Lett.* **B293** (1992) 430; (E) **B297** (1993) 477.
- [9] B. Blok and M. Shifman, *Nucl. Phys.* **B399** (1993) 441; 459.
- [10] J. Chay, H. Georgi and B. Grinstein, *Phys. Lett.* **B247** (1990) 399.
- [11] I. Bigi, B. Blok, M. Shifman, N. Uraltsev and A. Vainshtein, *The Fermilab Meeting* Proc. of the 1992 DPF meeting of APS, C. H. Albright *et al.*, Eds. World Scientific, Singapore 1993, vol. 1, page 610.
- [12] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Phys. Rev. Lett.* **71** (1993) 496.
- [13] B. Blok, L. Koyrakh, M. Shifman and A. Vainshtein, *Phys. Rev.* **D49** (1994) 3356.
- [14] A. Manohar and M. Wise, *Phys. Rev.* **D49** (1994) 1310.
- [15] T. Mannel, *Nucl. Phys.* **B413** (1994) 396.

- [16] M. Neubert, *QCD based interpretation of the lepton spectrum in inclusive $\bar{B} \rightarrow l\bar{\nu}$ decays*, Preprint CERN- TH.7087/93.
- [17] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *On the motion of the heavy quarks inside hadrons: universal distributions and inclusive decays*, Preprint TPI-MINN-93/60-T [*Int. Journ. Mod. Phys.*, to appear].
- [18] M. Neubert, *Analysis of the photon spectrum in inclusive $B \rightarrow X_s\gamma$ decays*, Preprint CERN-TH.7113/93.
- [19] A. Falk, E. Jenkins, A. Manohar and M. Wise, *QCD corrections and the endpoint of the lepton spectrum in semileptonic B decays*, Preprint UCSD/PTH 93-38.
- [20] M. Neubert, *Phys. Lett.* **B322** (1994) 419.
- [21] T. Mannel and M. Neubert, *Resummation of nonperturbative corrections to the lepton spectrum in inclusive $B \rightarrow Xl\bar{\nu}$ decays*, Preprint CERN-TH.7156/94.
- [22] M. Shifman, N. Uraltsev and A. Vainshtein, $|V_{cb}|$ from OPE sum rules for heavy flavor transitions, Preprint TPI-MINN-94/13-T.
- [23] N. Cabibbo and L. Maiani, *Phys. Lett.* **79B** (1978) 109;
these authors adapted QED radiative correction to $\mu \rightarrow e\nu\bar{\nu}$; the original QED calculations are published in R.E. Behrends, R.J. Finkelstein and A. Sirlin, *Phys. Rev.* **101** (1956) 866; S. Berman, *Phys. Rev.* **112** (1958) 267.
- [24] For comprehensive reviews see
V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, *Fortschr. Phys.* **32** (1984) 585;
M. Shifman, Ed., *Vacuum Structure and QCD Sum Rules*, North-Holland, 1992, Chapter 3.
- [25] G. Altarelli, N. Cabibbo, G. Corbo, L. Maiani and G. Martinelli, *Nucl. Phys.* **B208** (1982) 365.
- [26] M. Voloshin and M. Shifman, *Yad. Fiz.* **47** (1988) 801 [*Sov. J. Nucl. Phys.* **47** (1988) 511].
- [27] S. Nussinov and W. Wetzel, *Phys. Rev.* **D36** (1987) 130.
- [28] N. Isgur and M. Wise, *Phys. Lett.* **B232** (1989) 113; *Phys. Lett.* **B237** (1990) 527.
- [29] J. Bjorken, in *Proceedings of the 4th Rencontres de Physique de la Vallée d'Aoste*, La Thuille, Italy, 1990 ed. M. Greco (Editions Frontières, Gif-Sur-Yvette, France, 1990).
- [30] N. Isgur and M. Wise, *Phys. Rev.* **D43** (1991) 819;
J.D. Bjorken, I. Dunietz and J. Taron, *Nucl. Phys.* **B371** (1992) 111.
- [31] A. Bareiss and E. Paschos, *Nucl. Phys.* **B327** (1989) 353;
C. Jin, W. Palmer and E. Paschos, *A parton model for inclusive semileptonic B meson decays*, Preprint DO-TH 93/29.

- [32] C. Csaki and L. Randall, *Phys. Lett.* **B324** (1994) 451;
G. Baillie, *Phys. Lett.* **B324** (1994) 446.
- [33] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Heavy quark distribution function in QCD and the AC^2M^2 model*, Preprint TPI-MINN-94/2-T.
- [34] P. Ball and V. Braun, *Phys. Rev.* **D49** (1994) 2472;
earlier estimates can be found in M. Neubert, *Phys. Rev.* **D46** (1992) 1076; E. Bagan, P. Ball, V. Braun and H. Dosch, *Phys. Lett.* **B278** (1992) 457.
- [35] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *The pole mass of the heavy quark. Perturbation theory and beyond*, Preprint TPI-MINN-94/4-T.
- [36] V. Novikov, L. Okun, M. Shifman, A. Vainshtein, M. Voloshin and V. Zakharov, *Phys. Reports* **41** (1978) 1.
- [37] G. 't Hooft, *in The Whys Of Subnuclear Physics*, Erice 1977, ed. A. Zichichi (Plenum, New York, 1977), p. 943.
- [38] B. Lautrup, *Phys. Lett.* **69B** (1977) 109;
G. Parisi, *Phys. Lett.* **76B** (1978) 65; *Nucl. Phys.* **B150** (1979) 163;
A. Mueller, *Nucl. Phys.* **B250** (1985) 327.
- [39] For a recent review see
A. H. Mueller, *in Proc. Int. Conf. "QCD – 20 Years Later"*, Aachen 1992, eds. P. Zerwas and H. Kastrup, (World Scientific, Singapore, 1993), vol. 1, page 162.
- [40] M. Beneke and V. Braun, *Heavy quark effective theory beyond perturbation theory: renormalons, the pole mass and the residual mass term*, Preprint MPI-PhT/94-9;
M. Beneke, V. Braun and V. Zakharov, *On infrared effects in inclusive B decays: Bloch-Nordsieck cancellations beyond logarithms*, Preprint MPI-PhT/94-18.
- [41] I. Bigi, M. Shifman, N. Uraltsev and A. Vainshtein, *Sum rules for heavy flavor transitions in the SV limit*, Preprint TPI-MINN-94/12-T.
- [42] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Nucl. Phys.* **B147** (1979) 385; 448
[reprinted in M. Shifman, Ed., *Vacuum Structure and QCD Sum Rules*, North-Holland, 1992, p. 24].
- [43] G. 't Hooft, *Nucl. Phys.* **B72** (1974) 461;
E. Witten, *Nucl. Phys.* **B149** (1979) 285.
- [44] M. Shifman, *in preparation*.
- [45] T. Banks and A. Casher, *Nucl. Phys.* **B169** (1980) 103.
- [46] V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Uspekhi Fiz. Nauk* **136** (1982) 553 [*Sov. Phys. Uspekhi* **24** (1982) 195; reprinted in *Instantons in gauge Theories*, M. Shifman, Ed. World Scientific, Singapore, 1994]
- [47] J. Koplik, A. Neveu and S. Nussinov, *Nucl. Phys.* **B123** (1977) 109.
- [48] Z.X. Wang and D.R. Guo, *Special Functions*, World Scientific, Singapore, 1989.

Figure Captions

Fig. 1.

Different inclusive transitions of the heavy quark in the quark language. The closed circle denotes an effective electroweak vertex. Thick solid line denotes the decaying heavy quark.

Fig. 2.

Nuclear β decay.

Fig. 3.

Graphic representation of the transition operator in the inclusive semileptonic decay. The region inside the dashed line is governed by short-distance physics.

Fig. 4

The diagrams determining the coefficients of the operators $\bar{Q}Q$ and \mathcal{O}_G in the transition operator of Fig. 3.

Fig. 5.

The photon energy spectrum in the $Q \rightarrow q\gamma$ transition (the parton-model approximation). Due to the two-body kinematics the photon line is monochromatic. The final quark is assumed to be massless.

Fig. 6.

A realistic spectrum in the inclusive hadronic decay $H_Q \rightarrow X_q\gamma$. The kinematical boundary is shifted to the right of the parton-model line by $\bar{\Lambda}/2$. The final quark q is assumed to be massless.

Fig. 7.

Graphic representation of the transition operator in the problem of the end-point spectrum (the Born approximation). All quark lines are in the background (soft) gluon field.

Fig. 8.

Evolution of the spectrum of Fig. 6 as the mass of the final quark increases (the schematic plot refers to $m_q \sim m_Q/2$). The effects of the hard gluon bremsstrahlung are not included.

Fig. 9.

The photon spectrum in the SV limit ($m_Q - m_q \ll m_{Q,q}$). The effects of the hard gluon bremsstrahlung are not included.

Fig. 10.

The Roman model for the distribution function relevant to $b \rightarrow u$. The Roman *ansatz* has two fit parameters, the Fermi momentum p_F and the spectator mass m_{sp} . The parameter ρ is defined as $\rho = (m_{sp}/p_F)^2$.

Fig. 11.

Infrared renormalon in the pole mass.

Fig. 12.

A schematic plot of the structure function w_1 in the inclusive semileptonic decay $B \rightarrow X_c l \nu$ in the limit $|\vec{q}|/M_D \ll 1$. A long perturbative tail due to the hard gluon shake-off is depicted to the left of the primordial domain. The dashed line shows an extrapolation of this tail in

the region where $q_{0max} - q_0 \sim \overline{\Lambda}$.

Fig. 13.

Diagrammatic representation for the correlation function (46). (a) The product of the quark Green functions in the soft gluon field; (b) Building the operator product expansion: the light quark line is soft in the limit $m_q \rightarrow 0$ and can be cut.

Fig. 14.

A schematic plot of the spectral density for the correlation function (46).

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>

This figure "fig2-1.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>

This figure "fig1-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>

This figure "fig2-2.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>

This figure "fig1-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>

This figure "fig2-3.png" is available in "png" format from:

<http://arXiv.org/ps/hep-ph/9405246v1>